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Mathematical Visualization Process of Junior High School Students in Solving a Contextual Problem Based on Cognitive Style

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Abstract. The aim of this research was to describe the mathematical visualization process of Junior High School students in solving contextual problems based on cognitive style. Mathematical visualization process in this research was seen from aspects of *image generation, image inspection, image scanning*, and *image transformation*. The research subject was the students in the eighth grade based on GEFT test (*Group Embedded Figures Test*) adopted from Within to determining the category of cognitive style owned by the students namely field independent or field dependent and communicative. The data collection was through visualization test in contextual problem and interview. The validity was seen through time triangulation. The data analysis referred to the aspect of mathematical visualization through steps of categorization, reduction, discussion, and conclusion. The results showed that field-independent and field-dependent subjects were difference in responding to contextual problems. The field-independent subject presented in the form of 2D and 3D, while the field-dependent subject presented in the form of 3D. Both of the subjects had different perception to see the swimming pool. The field-independent subject saw from the top, while the field-dependent subject from the side. The field-independent subject chose to use *partition-object* strategy, while the field-dependent subject chose to use *general-object* strategy. Both the subjects did transformation in an object rotation to get the solution. This research is reference to mathematical curriculum developers of Junior High School in Indonesia. Besides, teacher could develop the students' mathematical visualization by using technology media or software, such as geogebra, portable cabri in learning.

INTRODUCTION

This research was inspired from Presmeg study about the ability in mathematical visualization or which was named with mathematical visualization. It was the process in understanding the question of the algebra and visual shape by using the students' ability in observing the pictures by using their eyes and then connected it to their brains to solve the mathematic problem. The mathematical visualization was a mental ability or a mental activity where individual improved the connection between an internal construct and something which accessed through senses [1]. Mathematical visualization was one of abilities which was developed in a formal operational stage (about 12 years old until adult), which meant they were first grade students in the senior high school [2].

The researcher was motivated with [3] which stated that it was very rare in conducting research about mathematical visualization. So, [3] recommended there would be further research and wider about mathematical visualization.

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Besides, in *New Brunswick Mathematics Grade 8 Curriculum Guide* mathematical visualization was one of essential components developed to reach the educational goal. So it became one of the reasons to choose the theme.

Mathematical visualization had an essential role in developing thought, understanding mathematics and in transition of concrete thought into abstract one which related to solve the problem [4]. I wanted to make sure that mathematical visualization was needed to solve mathematical problem. Therefore mathematical problem became one of essential means to develop students' ability in understanding mathematical concept and solved the mathematical problem.

According to Ollerton [5], stated that the ability in solving problems was one of essential aspects in self learning and helped to remove the learning system into educating system. In developing skill in solving mathematical problem had been admitted as essential goal in every educational level [6]. The ability in solving the problem was the heart of mathematics. The ability to solve the problem is one of the most important objectives in the study of mathematics [7], that the ability in solving the problem was one of the essential goals in mathematics study. The problem solving was unseparated from all mathematics studies [8]. Therefore the problem solving was needed to know by all students especially for the changing level from concrete thought level into the abstract one, since the students were in the first grade of senior high school.

One of terms which had a close relation with problem solving was how the process was in understanding and exploring information. [3] identified that one of abilities in exploring mathematics problem and for giving the meaning of mathematics concept related with the success in solving the problems, namely mathematical visualization. Based on opinions of "*particularly, in the context of mathematical problem solving, visualization refers to the understanding of the problem with the construction and/or the use of a diagram or a picture to help obtain a solution*" namely in the context of solving mathematics problem., visualization referred in understanding problem by constructing and/or by using diagram or picture to determine the solution [9]. That's why mathematical visualization had to be given more attention and developed in mathematics education and needed further investigation from the beginning in junior high school.

Various perception had by an individual in digging information was possibly to have exploration, one of someone's cognitive style. Visualization had an essential role in knowing and assessing different cognitive style (psychologically) and helped to master the technique to see the mathematical situation. In this research, the cognitive style was covered as *field independent* cognitive style [10], [11].

THEORETICAL FRAMEWORK

Visualization had an essential role in developing thinking process and understanding mathematics and as transition from concrete thought into the abstract one in solving the problem. Some research which discussed the benefit of visualization related to solving mathematical problem [1], [11], [12]. Besides, New Brunswick Mathematics Grade 8 Curriculum Guide (2011) stated that visualization was one of essential components which became the mathematics education goal.

In general in some of research had been discussed about visualization and the connection. According to [13] in defining "visualization", there were so many findings such as visualization, visual literacy, and image which were often applied and could be changed through literature. Mathematical visualization was a mental ability or a mental activity where an individual improved the connection between an internal construct and something which accessed through senses [1]. Mathematical visualization was an ability to image, change, generalize, communicate, prove, imagine visual information, which played main role in understanding picture/visual [14]. Visualization was a process in forming image through an action where an individual formed strong relation between thought with something accessed through senses [15]. Visualization was one from three free cognitive styles which fulfilled specific epistemology function in mathematics, besides construction and thought [16].

Next [17] stated that: "Visualization is an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the senses. Such a connection can be made in either of two directions. An act of visualization may consists of any mental construction of objects or processes that an individual associates with objects or events perceived by her or him as external. Alternatively, an act of visualization may consist of the construction, on some external medium such as paper, chalkboard or computer screen, of objects or events that the individual identifies with object (s) or process (es) in her or his mind"

In solving mathematical problem, there were some aspects in visualization process such as: 1) generation,to explore information about problem items; 2) inspection, to choose some certain mathematical idea in solving problem; 3) transformation, to change or to manipulate the image of mathematical idea in solving problem; and 4) use, to make

representative forms in solving problem [18]. While, according to [13] visualization process were: 1) understanding,to understand the relation of items presented in the problem; 2) connecting, to connect some knowledge which was owned in solving the problem; 3) constructing,to construct a visual representation (both in mental and in external such as paper or through technology tools); 4) using, finishing the problem by using visual representation; and 5) encoding, to give a code for the answer. Based on the case, mathematical visualization was based on some following aspects: generating, inspecting, transforming, and evaluating.

Based on the explanation above about the visualization, the definition of mathematical visualization was mental activity or an individual cognitive activity in exploring and manipulating as creativity during finishing the problem with the aim of communicating information, thought and developed the idea known before to get higher understanding. However, the focus in this research was a mental activity in developing information and idea by using process of image generation, image inspection, image scanning, and image transformation in solving contextual problem. Image generation which is the ability to form visual image, requires activating stored visual information and using it to create a pattern in a spatial short-term memory structure. Image inspection is the ability to interpret a pattern in the visual buffer as depicting an object or part of the object. Image scanning the systematic shifting of attention over an imaged, plays a critical role in this ability. Image transformation is the ability to rotate or otherwise alter an imaged pattern. The explanation of each mathematical visualization aspect in solving the contextual problem presented in table 1 as the following.

TABLE 1. Aspects in Mathematical Visualization Process

Aspects in Mathematical Visualization Process	Sub Aspects in Mathematical Visualization Process	Description
Image generation	Exploring the visual ideas	Mental activity done by the subject in identifying the elements represented in the forms of 2D and 3D
	Recognizing the visual ideas	Mental activity in recognizing and connecting the objects based on perception and previous knowledge
Image inspection	Examining the visual object	Mental activity in observing the elements of object which were made thoroughly or partly
	Choosing visual ideasl	Mental activity in determining the strategy in solving based on the result of the previous observing object
Image scanning	Elimination visual ideas	Mental activity in determining to eliminate the visual object based on strategy applied
Image transformation	Changing visual object	Mental activity in changing the object by rotation

Next, cognitive psychology had two meanings in various mental activity in individual mental and referred to the theoritical psychology approach such as mental process and human knowledge. The thought which involved getting information, transforming and using information where had an aim to image something such as individual mental process to get knowledge. Gender was one of important role in getting until using information. There were physical differences between male's brain and female's. The different structure could become a factor which differed the behavior, development and cognitive process between male and female.

Solving problem was a cognitive process which needed effort and consentration, because to get the answer from the problem, it should be held from valuable information in memory or previous knowledge. When the students did solving problem activity, the students were possible to us different approaches in solving the problem. It occured because each students had different characteristics. The difference was different cognitive style. Aspect of cognitive

style had an essential role during mathematical process mainly about receiving information until using information in finishing problem [10], [11].

In this research, the case study was seen by the researcher to explain the mathematical visualization process in male students based on cognitive style. The indicators used in knowing the mathematical visualization process for eighth grade students in solving contextual problem as the following.

METHOD

Subject

The subjects of this research were two male students of Junior High School in the eighth grade, where there is a student who had cognitive field independent style and a student who had cognitive field dependent style. The selection of the male subjects were by considering research [11] and [4] that male students were better than the female ones in processing of information, various spatial tasks, and visual-spatial thought. I used random sampling to the male students. I only took two male students to represent the data because the male students almost had the same capability in visualization. It made me easier to explain and to describe their ability in visualization when I only used two subjects. I was sure that it would be more efficient.

Instrument

The instrument in this research was:

- Giving GEFT test (*Group Embedded Figures Test*) adopted from within to determining the category of cognitive style owned by the students namely field independent or field dependent.
- The sheet of test of mathematical visualization, where the test content was about contextual problem showed on the table 2 as the following.

TABLE 2. The Sheet of Test of Mathematical Visualization

A swimming pool which had a size of length 50 meters and width 25 meters which the depth was divided into two categories, namely adult and teenager category. The adult one was with the depth of 3 meters with the distance in 30 meters and the teenager one with the depth 1,5 meters with the distance in 17 meters, while the bottom barrier was made in the form of ramp. How many liters of water required to fill the pool? Explain your answer!

Procedure

The research design was descriptive explorative research by using qualitative approach. The data collection and data analysis were based on [19]. The data collection procedure was aimed at exploring the mathematical visualization process in solving contextual problem, started by giving mathematical visualization test to the subjects, and then they did it based on their cognitive style and they wrote their answer on the answer sheet. After that, the researcher did the depth interview based on mathematical visualization aspects referred to the answer. The credible data was used by time triangulation. The analysis data referred to the mathematical visualization aspect through the steps of categorizing data, reducing, discussing, and making conclusion about the mathematical visualization process.

RESULT

Mathematical Visualization Process to the Field-Independent Student

Image Generation Aspect

The student classified the items such as the length entirely namely 50 meters divided into categories namely adult and teenager, each length of categories is 30 meters and 17 maters, the width is 25 meters, and the depth was from each categories is 3 meters and 1,5 meters, and the item which is questioned. The student represented the swimming pool in the form of 2D and 3D, such as rectangle, , triangular, cube, and triangular prism. To make it easier in understanding the picture which was made, the subject showed the perspective of the swimming pool from the up and bottom position. The student explored the shape of the swimming pool shown on the Fig. 1 below.

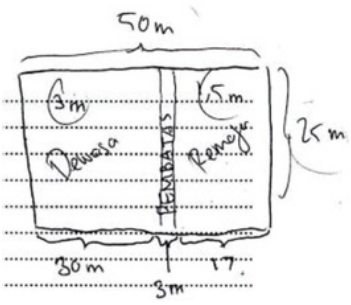

FIGURE 1. Representation of swimming pool by the Field-Independent Student

On Fig. 1 above, the student gave the additional information *"bracket"* showed the size of the length and the width of the swimming pool. The subject also gave explanation "BARRIER" which showed the area between both categories of the swimming pool. For stating the size of the depth, the subject gave the circle sign on the number "3m" and "1,5m". This was the interview between the researcher and the subject.

The researcher	: why did mas Haikal draw the shape of the swimming pool like this?
The subject:	: in my opinion, it would be easier to understand, such as the entire length of the swimming pool 50 meters, the width was 25 meters, the swimming pool was divided into 2 categories namely adult and teenager, the length of adult category was 30 meters, the length of teenager category was 17 maters, then the depth for adult was 3 meters and 1,5 meters for teenager.
The researcher	: what did you mean with being easier to understand?
The subject	: I thought it would be easy to draw it, then I drew the surface of it from the up part *(representation of the swimming pool I 2 D)*, then for the depth I only imagined it because I did not see this picture.
The researcher	: How did mas Haikal imagined it?
The subject	: Yes, Imagined the depth side from the sides of the swimming pool was same with the sloping area between two categories of the swimming pool.

Besides, in the contextual problem, the subject developed the imaginative information based on the situation which was ever happened. The subject related it with the previous knowledge such as characteristics of plane and geometry. The subject knew the items of the size in the rectangular, cube, and triangular prism connected with the parts of the swimming pool. The subject represented the swimming pool based on the statements in the question. The subject

categorized the data based on the size and compared the characteristics from each of the plane and the geometry of the swimming pool.

Image Inspection Aspect

The next process done on the mathematical visualization was namely inspecting picture. The subject observed the swimming pool based on the categories, so that the subject parted the pool became some parts such as the area of the swimming pool for the adult category, the area of the swimming pool for adult and the area of the swimming pool for the teenager and the area between the two categories. In each area, the subject saw the different item measured, so that the subject determined some planes such as cube, and triangular prism. The subject stated that the area of the swimming pool for the adult category was the cube because the size among the length, the width, and the depth from different swimming pool. And also the area of the swimming pool for teenager category, the subject observed the sloping area so that the construction of the triangular prism. This was the interview between the researcher and the subject.

The researcher	: why did mas Haikal use the pattern of cube?
The subject	: yes, **I showed the different size**, the length, the width, and the height. The length was 30 meters, the width was 25 meters, and the height was 3 meters so that the pattern of the cube was for the different edges.
The researcher	: oo..besides the different size, why was mas Haikal sure that this was the plane *(while pointing at the swimming pool for the adult)*
The subject	: I imagined because **it was different equilateral..front equilateral** *(while practicing the related front equilateral)*…wasn't it noticeable.. *(while practicing the equilateral side)* and the up equilateral *(while pointing at the answer sheet)*
The researcher	: ooo, then mas Haikal stated that counting this volume of the pool first *(the researcher meant with the adult swimming pool)*?
The subject	: I counted each swimming pool category, like **I broke the swimming pool**…to make it easier, and then I used the pattern of the cube the length times the width times the height *(the subject of LFI pointed at the answer sheet)*…. And this p was the length of the pool, this l was the width and t was the depth of the pool

In this aspect, the subject noticed the several sizes as the reference in choosing strategy. The subject used the *object partition strategy*, where the subject counted the volume from each of the swimming pool. In the swimming for the adult, the subject noticed the different equilateral, and also the swimming pool for the teenager category. In the swimming pool for adult and teenager, the subject used the cube pattern, namely *"volume of the swimming pool for adult/teenager = $p \bullet l \bullet t$"*.

Image Scanning Aspect

The subject simplified the visual ideas from the complex form to solve the contextual problem. The subject moved the picture to ease the solution, such as separating the barrier between both categories in the swimming pool. The form of the movement from the picture shown in the Fig. 2 below.

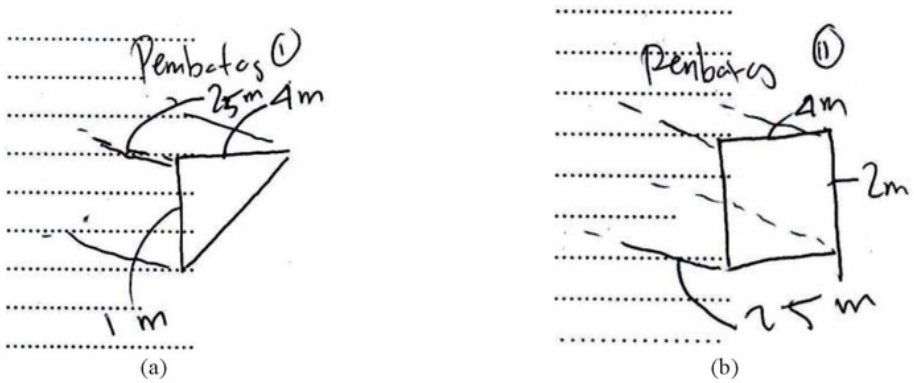

FIGURE 2. (a) Movement process of bottom barrier area, (b) movement process of bottom barrier area

The beginning movement done by the subject meant that he took the bottom from the barrier area. And then continued the up part from the barrier area of both categories of the swimming pool. In the bottom part, the subject assumed the triangular prism with the sleeping position, while, the up part of barrier, the subject assumed the cube with sleeping position. This was the interview between the researcher and the subject.

The researcher	: try to explain this picture!
The subject	: from the 1 barrier, the flat part was the length of the barrier…not the width of the pool, 1 meter which was erect in 1 meter, then the tilt part…the entire width was 25 meters.
The researcher	: why did the picture parted from the others?
The subject	: yes, I took the picture so that it would be easy to see it (*the subject meant that it was related with the barrier area of both categories of the swimming pool*)
The researcher	: if it was seen from the picture *(the researcher meant with Fig. 2a)* how was the position?
The subject	: eee…it was seen from its side,
The researcher	: seen from its side, what did it mean?
The subject	: tilt…sleeping…
The researcher	: ooo..sleeping
The subject	: yes
The researcher	: okay, you stated that *(while pointing at Fig. 2a)* the shape of prism ya..
The subject	: yes
The researcher	: what prism is this mas?
The subject	: triangular prism
The researcher	: why was the triangular prism?
The subject	: because the bottom part of this was sloping, so the basic of this prism was in the shape of triangular seen from its side

Then, on the Fig. 2b, the subject took the up part from the barrier, so that the subject got the cube with position "sleeping". This was the interview between the researchers with the subject.

The subject : the second barrier was in cube shape…the flat plat *(the subject pointed at the flat part in Fig. 2b)* the length from ee…the barrier, not the width of the pool but the length of the barrier, was 4 meters, the erect length was from the high part from these two barriers *(the subject pointed at the erect part in Fig. 2b)* the 2 meters, then in the sloping part *(the subject showed the dropping out lines in the part of the Fig. 2b)* 25 meters, the entire width of the pool.

The researcher : emm, could you explain the relation of barrier 1 (*the researcher pointed at Fig. 2a*) and the barrier 2 *(the researcher was pointing at Fig. 2b)*?

The subject : the barrier 1 was the bottom part, the barrier 2 was the top one, so actually the barrier 1 and barrier 2 were coincide

The researcher : ooo, coincide

The subject : yes, but I differed it into 2 so that it was easy to count it

The researcher : from this picture *(the researcher pointed at the Fig. 2b)*, was this the position?

The subject : the sloping and sleeping position such as the picture of the bottom part *(referred to the Fig. 2a)*

The process of the picture movement was done by the subject showed about simplifying the picture, where the subject took the essential picture was from the complex picture to ease the solution. The subject did the picture movement and the picture partition. The movement done by the subject noticed the position from the object, so that the new object formed based on the beginning of object perception.

Image Transformation Aspects

For the barrier of both categories of the swimming pool, the subject did rotation on the picture. The beginning of rotation was done on the bottom object from the barrier, where the subject rotated vertically from the triangular prism for making it easy in counting. In the beginning object, the subject had not determined the units, however after the rotation, the subject was able to determine the units. Some units were expressed by the subject, namely the prism base was the triangular form, where it was from the sloping area seen from its side, and the width of the swimming pool was changed to be higher from the prism. This was the interview between the researchers with the subject.

The researcher : what did you do to get the volume of barrier area?

The subject : I counted area 1 (*referred to the Fig. 2a*), where the picture I rotated so that it would be easier to count it.

The researcher : easy, what did you mean?

The subject : yes, with the erect position, I could see t22ase of the triangular and the wide pool was the height, so the volume counted was the area of the base times the height

After that, the subject rotated the object represented the top part of barrier. The object from the top part was cube, where the cube was in "sleeping" position like in the picture *2b.* To make it easy to solve it, the subject rotated vertically, so that the cube was in the erect position. This was the interview between the researcher and the subject.

The researcher : how did you look for this volume? *(the researcher pointed at the picture 2b)*

The subject : it was like in this picture *(the subject meant with the picture 2a),* I rotated the cube, so that I could get the cube base was from the side and the height of the cube was the width of the pool.

The researcher : why did it have to be rotated?

The subject : to make it easier to do it, to determine the base and the height

To suit with the units needed, the subject changed the length into units of volume. This was the interview when the subject changed the unit of length into the unit of volume of the swimming pool in adult category.

The researcher	: how was the result?
The subject	: 2250 meter cubic then I changed it into liter
The researcher	: how come?
The subject	: because one meter cubic was same with 1 kilo liter, then I decreased it from *meter* into *dm* ... so that 2250 meter cubic was 2.250.000 liter
The researcher	: what did you mean with decreasing into 1? How could it be that there were three zero?
The subject	: yes, in the ladder…from *meter* to *dm* if it went down into one, it would be multiplied with 10, so because it was in 3 steps and it was multiplied with 1000
The researcher	: oo…that's it, so how much was the result?
The subject	: yes, the volume of adult swimming pool was 2.250.000 liter

The unit got from the subject for the final result was in m^3, while the unit wanted was in liter so that the subject converted the length unit into the volume unit. The subject related it with unit of ladder, namely 1 m = 10 dm and 1 m^3 = 1 kilo liter, and 1 dm^3 = 1 liter, so 1 m^3 = 1000 liter.

Based on the counting result, the subject got the volume of the swimming pool for adult category was 2.250.000 liter, the volume for the teenager was 637.500 liter. While, the volume of the barrier of the two categories in the bottom was 56.250 liter, and the volume in the top was 112.500 liter. The subject expressed that the all result was then accumulated and got the final answer 3.056.000 liter.

Mathematical Visualization Process to the Field-dependent Student

Image Generation Aspect

The subject represented the swimming pool was in the form 3D. The subject identified the units of the swimming pool such as the length of the swimming pool was 50 meters, the width was 25 meters for each category. The subject explored the shape of the swimming pool from the above point of view. In the swimming pool, representation in the arrow sign to show the size of the swimming pool in the adult and teenager category. Besides, the subject also added writing in the swimming pool for teenager category. The result of exploration about the swimming pool was shown in Fig. 3 as the following.

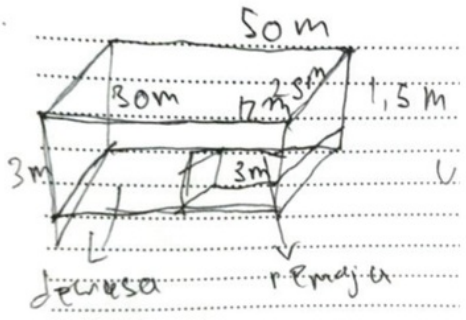

FIGURE 3. Representation of Swimming Pool by the Field-Dependent Student

To make it clear in Fig. 3, this was the interview between the researcher and the subject.

The researcher : Rayhan, please try to explain this picture *(Fig. 3)*

The subject : first, I made the pool in the shape of cube, then I made the barrier *(while showing Fig. 3)*, because the shape was sloping so I made it tilt..for this part

The researcher : why did you make the cube?

The subject : to make it easier to see the shape

The researcher : what did you mean?

The subject : so, I thought the whole pool, although there was sloping side, because we saw from outside, it looked like a surface of cube, and the bottom was like a trapezium

Besides, in picture 3, the subject explained that exactly in the pool units, such as 3 meters showed the depth of the swimming pool for adult category, 1,5 meters showed the depth of the swimming pool for teenager category, 30 meters showed the length of the swimming pool for adult category, 17 meters to show the length of the swimming pool for teenager category. However, in the length of both categories, the subject gave explanation on the bottom area of the swimming pool for teenager category.

Image Inspection Aspect

The subject chose the problem solving which was suitable for the image made, where the subject determined the entire volume of the swimming pool in the pattern of cube. The subject stated that the length referred to the entire swimming pool namely 50 meters, the width referred to the width of the swimming pool 25 meters, and the height referred to the depth of the swimming pool was 3 meters. The subject used the depth 3 meter as the height, because the subject assumed the depth of the swimming pool between the adult and teenager category was same. This was the interview between the researcher and the subject.

The researcher : how would mas Rayhan determine the volume of the swimming pool?

The subject : in the beginning, I counted the swimming pool, I thought the swimming pool was in cube because the size was different, then I counted the volume in the bottom part of the swimming pool in the teenager category, and later I decreased the first volume with the volume of the bottom part of the swimming pool for teenager category.

The researcher : why did mas Rayhan look for the volume of the bottom part of the swimming pool for the teenager category?

The subject : yes, so that it would be easy to know the volume of the pool, this was the shape of trapezium so I knew the volume.

The researcher : where did get the trapezium? And how did you count it?

The subject : I saw it from the front (*while pointing at picture 3*) then I counted the wide trapezoidal then I multiplied it with the height (*the subject pointed at the width of the pool in Fig. 3*)

Determining the strategy which was done by the subject showed that there was a perception about the shape of the swimming pool seen from outside. The subject used the strategy *general-object*, where the subject counted the entire volume of the object (cube) with the length to show the size for the whole of swimming pool, the width showed the width of the swimming pool and the height showed the depth of the swimming pool for adult category. And after that, the subject looked for the difference between the volume of the object (cube) with the volume of the bottom part of the swimming pool for teenager category. Besides, the subject identified that the different size became the reason why he used the pattern of cube.

Image Scanning Aspect

To make it easy to count the volume of the object of the swimming pool, the subject moved the certain object. The subject made the prism as the bottom part of the swimming pool named ***"empty space"***. This object representation on the swimming pool base was from the teenager category shown in the Fig. 4 as the following

(a) (b)

FIGURE 4. (a) object area in the bottom of the pool for the teenager category, (b) object area in the bottom of the pool for the teenager category with the size.

Representation shown in Fig. 4a, the subject made the object which showed the pat in the bottom part of the swimming pool for the teenager category. In picture 4a, it showed the prism, where the subject expressed the side of it was trapezoidal bracket. While in picture 4b, the subject gave the explanation about the size from units of the object, such as 1,5 was in the height (depth) of the prism, 25 was the width of the pool, 17 was the length of the swimming pool for teenager category, and 20 which showed the length of the bottom part of trapezium, where the subject added the sides of trapezium with the height of barrier of the two categories, namely 17 + 3 = 20.

Image Transformation Aspect

In Fig. 4a or 4b which were called "empty space", the subject did the object rotation to get the volume. The subject did the rotation vertically. The subject explained that the base of the prism was trapezoidal bracket, and the height of the prism was the width of the swimming pool. So that the subject used the prism pattern to determine the volume. This was the interview between the researcher and the subject.

The researcher : how did you get the area? (*Fig. 4b*)

The subject : I rotated the picture (*Fig. 4b*), so that the base now was trapezoidal bracket. Then, I counted the area of the trapezoidal bracket with the pattern of side in the bottom added by the top one and times the height and then divided by two.

Based on the subject's response, the base of the object "empty space" which was in the beginning was rectangular, changed into trapezoidal bracket because the subject did the object rotation. To determine the volume of "the empty space", the subject used the area pattern (*trapezoidal bracket)* multiplied with the prism height. This was the interview between the researcher and the subject.

The researcher : how did you determine the volume of this plane? *(Fig. 4a/4b)*

The subject : I multiplied it with the base area of the prism, namely 20 plus 17 times 1,5 and times 25 then divided by 2, so the volume I got was 693,75 m^3

The researcher : so, how much was the final answer?

The subject : for the entire volume (*swimming pool*) 3.750 m^3, then because it was asked in a liter, so I changed in the form of liter with the way $1m^3 = 1000\ dm^3$, then $1\ dm^3 = 1$ liter, so 3.750 times 1000 was 3.750.000 liters. And also the volume, (*the subject meant with the volume 693,75* m^3), I also multiplied it 1000, so it was 693.750 liters. Then, (the whole volume = 3.750.000) I reduced it with this (the volume of empty space = 693.750 liters), the result was 3.056.250 liters.

The subject looked for the difference between the beginning volumes determined by supposing that the pool was a cube with the volume of the object which was called "empty space". Besides, the subject changed the size of the unit from the result to be suited with the question. The way which the subject used was to change the unit into the length unit then converse it into the volume unit.

DISCUSSION

In solving contextual problem, there was difference in process of mathematical visualization in the image generation aspect between the subjects of field-independent with subjects of field-dependent. The subject of field-independent explored the information by representing in the form of 2D, where the representation which was got by the subject who saw the swimming pool from the top. Besides, the subject imagined the shape of the swimming pool was in 3D which was seen from the pool sides. The subject identified the object terms, such as the rectangular to represent the swimming pool seen from the top, the cube was to represent the swimming pool for the adult category and the teenager seen from the sides, the triangular prism was used to represent the barrier between the two categories seen from its sides. The subject also identified the different size of edge and side of the object. While, the field-dependent subject explored the information by representing in the form of 3D, where the representation which was got by the subject by seeing the swimming pool from the side. The subject assumed that the swimming pool was a cube and it was given a sign "empty space". Both of the subjects explored the problem based on the situation and the experience happened in the daily life.

Choosing the strategy by the two subjects was different, because it was influenced by the different perception in processing information. The field-independent subject observed the swimming pool from each part, where the subject saw the swimming pool for the adult and teenager category was cubes with certain sizes, and the plane among both categories was prism which had certain size. So the field-independent subject chose the strategy of *object partition*, where the strategy was determining the volume, and after that, it was accumulated to get the volume of the swimming pool. While the field-dependent subject saw from the whole side, where the swimming pool was drawn as the cube. After that, the subject identified which was not the part of the swimming pool which was called "empty space". The subject did not differ the swimming pool based o the category. The subject used the strategy *general-object,* where the subject determined the volume of the swimming pool entirely, and then looked for the volume of the pool with the volume of "empty space".

The movement of picture done by the field-independent subject referred to the complicated object, where the subject moved the barrier between the two categories separately. The subject represented the object into two parts, namely part1 was the plane which showed the bottom part from the barrier with the form of triangular prism, and part 2 which showed the bottom part from the barrier with the form of cube. While, the field-dependent subject made the picture which represented the part of the swimming pool which was called "empty space". The picture showed that the subject separated the position in the beginning.

The two subjects did rotation on the object. The subject of field-independent rotated the object in part 1 showed the base of triangular prism. Rotation process which was done by the field-independent subject vertically, where the beginning position from the object was horizontal changed into vertical, so there was a change of base of the object based on the subject perception. And also for the part 2, the subject also did the same thing such as in part 1. While, the subject of field-dependent did rotation to the object called "empty space", where the subject rotated the part vertically.

Based on the statement above, there was different response between the field-independent subjects in solving the contextual problem. The field-independent subject related the previous knowledge to explore the information. The subject noticed the position in representing the swimming pool. Besides, the subject imagined the shape which was not there in the answer sheet. While the field-dependent subject did not imagine based on situation happened. It was suitable with the findings of [18] and [2]. Representation showed to the different both subjects, where the field-independent subject represented in 2D while the field-dependent subject in 3D. The difference in observing the visual object done by the subject to influence in using the solving strategy. The field-independent subject counted the volume for each plane, namely volume of the swimming pool for adult category, volume of the swimming pool for teenager category, volume of the swimming pool for barrier category. However, the field-dependent subject did not look for the volume of each category, but determined the volume of the swimming pool entirely without parting them for some parts. It was same with the finding of [10].

The intensity of changing the visual object for the mathematical visualization process shown by both subjects was to ease the problem solving. It was based on the finding of [13]. For further research [9], showed that the contextual problem solving, mathematical visualization appeared for understanding the problem with the aim of making it easy to construct his understanding in the form of picture, pattern to help to get solution. Both of the subjects showed the different final result (solution). The field-independent subject was able to get the exact result, while the field-dependent subject got the uncertain result.

The researcher suggested the developer of educational curriculum in Junior High School Indonesia was to develop the ability of mathematical visualization mainly about geometry. It could be done by focusing on the contextual problem, because it could develop the ability in interpreting, representing, creating, manipulating visual ideas. Besides, teacher could develop the students' mathematical visualization by using technology media or software, such as geogebra, protable cabri, etc.

CONCLUSION

There was a difference in mathematical visualization between the field-independent subject and the field-dependent subject. The field-independent subject presented in the form of 2D and 3D, while the field-dependent subject presented in the form of 3D. Both of the subjects had different perception to see the swimming pool. The field-independent subject saw from the top, while the field-dependent subject from the side. Choosing the strategy was shown by each subjects from the perspective of the object made. The field-independent subject used the strategy of the *partition-object*, while the field-dependent subject used strategy of *general-object*. Both of the subjects did the object movement to make it easy to be solved. With the same way, both of the subjects did transformation in an object rotation to get the solution. The most important thing was, there was mathematical visualization implemented to the curriculum of Junior High School in Indonesia in solving the problem.

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